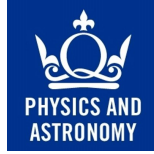# Cosmic Ray Muons Research Project

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Abstract

Muons are fundamental particles similar to electrons which can be created by cosmic rays bombarding the top-of-the-atmosphere. These muons can be detected on the ground using plastic scintillator, a photo-multiplier tube and a discriminator. Since muons are unstable particles, this setup allows the measurement of the mean lifetime of muons - an important quantity in a number of topics such as testing special relativity or calculating the strengths of the Weak and Higgs interactions. Using the equipment, you will conduct a research project by initially measuring the lifetime of muons from cosmic rays followed by your own independently motivated research.

1   Introduction

Muons $\mu^{\pm}$ are elementary particles (leptons) similar to the electron but with greater $(207\times)$ mass. On Earth, most naturally occurring muons are created by **cosmic rays** - particles (mostly protons) from somewhere in the galaxy which have been accelerated to extremely high energy by still unknown mechanisms. When a cosmic ray proton impacts atomic nuclei in the upper atmosphere, nuclear reactions occur which sometimes result in the creation of pions $\pi^{\pm,0}$ (a type of meson). These pions decay extremely quickly into so-called "secondary muons" and muon neutrinos/antineutrinos ν_{μ} $\bar{\nu}_{\mu}$ as shown in Figure 1.

The "secondary muons" continue in about the same direction as the original cosmic ray proton at a velocity near the speed of light, and can be detected on the ground. Muons, however, are unstable particles and will decay into an electron/positron of the same charge as the muon and two neutrinos of different types.

> **Exercise:** Draw all possible decay paths of both positive and negative muons. Be sure to follow all the conservation rules for fundamental particle interactions.
>
> Almost 100% of all muon decays are of the form
>
> $$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$
> $$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$
>
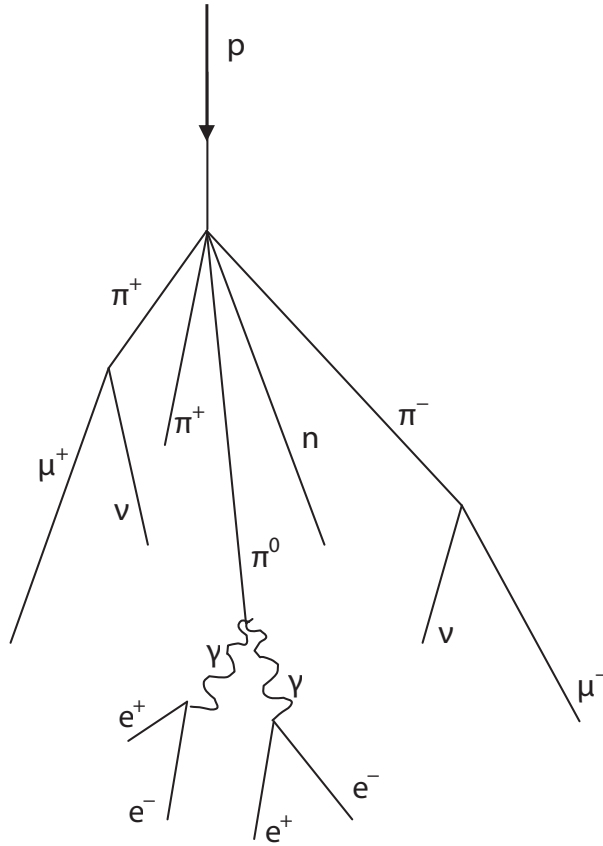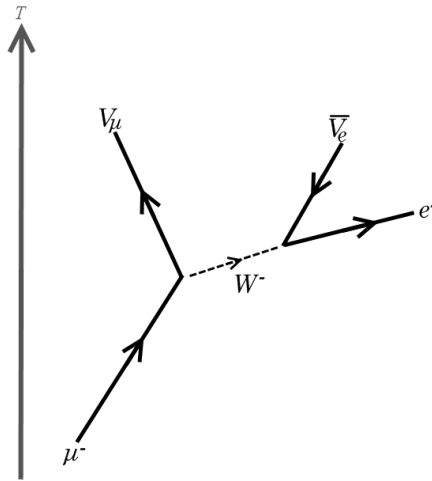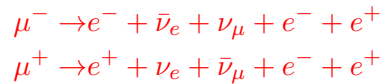> Alternatively the equivalent Feynman diagrams with $W^{\pm}$ boson e.g.

Figure 1: Cosmic ray cascade induced by a cosmic ray proton striking an air molecule.

However, more rare muon decays have been observed such as radiative muon decay, releasing an additional photon, which occurs in $1.4\pm0.4\%$ of decays for $E_\gamma > 10$ MeV

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu + \gamma$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \gamma$$

Furthermore muons have been seen to decay with an additional electron positron pair in $3.4 \pm 0.4 \times 10^{-3}\%$ of events, allowed because the muon mass is sufficiently large,

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu + e^- + e^+$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + e^- + e^+$$

For more see http://arxiv.org/pdf/hep-ph/9909265v1.pdf

Similarly to the radioactive decay of nuclei, the decay of muons into electrons/positrons typically occurs over some **mean lifetime** τ_μ. The muon lifetime is an important quantity: for instance it can be used to validate Einstein's theory of Special Relativity or to ascertain the coupling constant of the weak interaction and the vacuum expectation value of the Higgs field.

More generally, through precise measurements of all the properties of the known fundamental particles it is hoped that we might be able to answer some of the unknown questions in particle physics including: why is there more matter than antimatter, what makes up dark matter, are neutrinos their own antiparticle (majorana particles), do super-symmetric (SUSY) particles exist. In this research project you will initially be measuring the lifetimes of muons from cosmic rays. While this quantity is known to high precision already, whenever using new scientific instrumentation it is important to test its capabilities by replicating a known result. Once you have done this, you can continue to use the detector for your own independently motivated research.

2   Equipment

When charged particles travel through certain transparent materials, they can excite and/or ionise the bound electrons resulting in a flash of light when they de-excite/relax. This process is known as **scintillation** and it is a mechanism by which the particles can deposit some of their energy. In the equipment a plastic scintillator is used and the flashes are detected by a **photomultiplier tube** (PMT), an extremely sensitive light detector which converts the brief flash of light to a detectable pulse of electrons. Figure 2 shows a top down view of the PMT-Scintillator detector.

Cosmic ray muons typically have energies of ∼2 GeV, and pass right through the scintillator. In doing so they cause some ionisation, depositing ∼50 MeV in the scintillator - some fraction of this energy gets converted to photons of light, and some fraction of that light reaches a photomultiplier tube. Ambient beta and gamma rays have an energy of ∼1 MeV or less and a **discriminator** can be used to filter out the weak light pulses they create. So the scintillator/photomultiplier assembly, when properly configured, produces several electronic events every second, almost all of which are due to muons passing through the scintillator.

The truly exciting fraction of those events, however, is due to low energy muons who deposit the last of their kinetic energy, typically ∼50 MeV, in a scintillation flash **coming to rest inside the detector**. These muons will then go on to decay into electrons/positrons in the detector, thereby producing a second scintillation flash. By measuring the time difference between these two flashes for many muon events, we can estimate the mean lifetime of these particles (see section 3).

You will be doing precisely this using the following equipment:

- **PMT-Scintillator** - Muons are detected by interacting with detector and produce light in the scintillating material which is amplified by the photomultiplier tube and outputs an analogue signal. Figure 2 shows a top view of the detector.
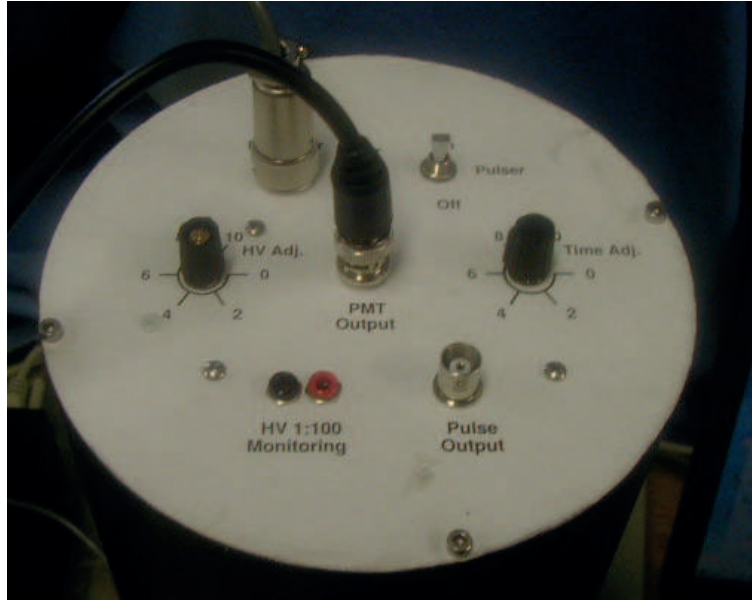
Figure 2: Top view of the PMT-Scintillator detector.

- **Muon Physics** - Detector Electronics, converts the analogue signal into a logic output in order to be read by the PC.

- **Oscilloscope** - Outputs a signal from the PMT and the discriminator, these should have a similar form to that seen in figure 3.

- **PC** - Used for data acquisition and analysis.

- **Cables** - to connect equipment together.

3   Theory

The decay of a single muon is a **Poisson process**. This means that the (differential) probability that a muon will decay dp during a certain (infinitesimally) small time interval between time t and $t + dt$, is constant

$$dp = \lambda dt \tag{1}$$

and importantly independent of the muon's history i.e. when it was created. Here λ is the decay constant of muons.

> **Exercise:** Show that if we denote the probability that the muon has not decayed until time t as $P(0, t)$ then the probability that the muon hasn't decayed by time $t + dt$ is given by $P(0, t)(1 - \lambda dt)$

$$Prob(\text{not decayed by } t + dt) = Prob(\text{not decayed by } t) \times Prob(\text{not decayed in interval } dt)$$
$$P(0, t + dt) = P(0, t) \times [1 - Prob(\text{decayed in interval } dt)]$$
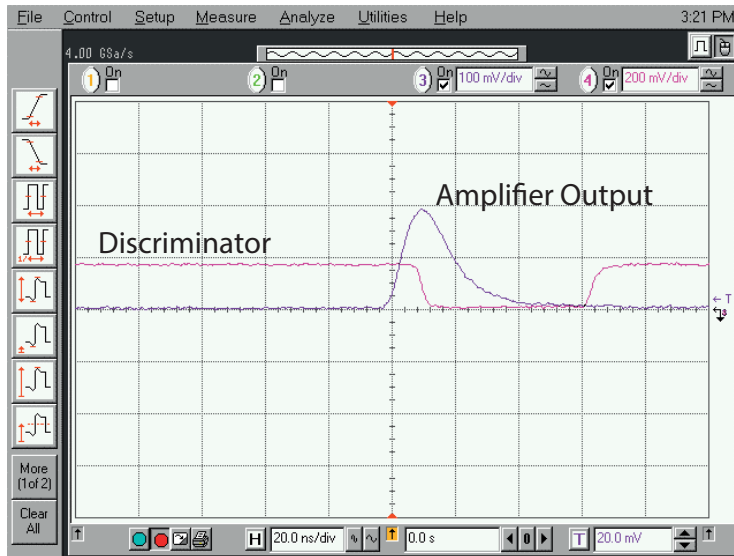$$P(0, t + dt) = P(0, t)(1 - \lambda dt)$$

Figure 3: Expected view from oscilloscope.

Students may also wish to draw a probability tree.

We therefore have that

$$P(0, t + dt) = P(0, t)(1 - \lambda dt) \tag{2}$$

Exercise*: Show that in the limit that dt goes to zero this becomes the differential equation

$$\frac{dP(0, t)}{dt} = -\lambda P(0, t) \tag{3}$$

$$
\begin{aligned}
P(0, t + dt) =& P(0, t)(1 - \lambda dt) \\
=& P(0, t) - P(0, t)\lambda dt \\
P(0, t + dt) - P(0, t) =& -\lambda P(0, t)dt \\
\frac{P(0, t + dt) - P(0, t)}{dt} =& -\lambda P(0, t) \\
\frac{dP(0, t)}{dt} =& -\lambda P(0, t)
\end{aligned}
$$

Let us call the time that the muon entered the detector t_1. Since we are only considering muons that cause a scintillation flash at time t_1 we require $P(0, t_1) = 1$.

Exercise*: Show that the solution to equation 3 is therefore given by

$$P(0, t) = e^{-\lambda(t - t_1)} \tag{4}$$

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*

Requires knowledge of calculus (differentiation/integration) from A-Level Maths

$$\frac{dP(0,t)}{dt} = -\lambda P(0,t)$$

$$\frac{dP(0,t)}{P(0,t)} = -\lambda dt$$

$$\int \frac{dP(0,t)}{P(0,t)} = \int -\lambda dt$$

$$\ln|P(0,t)| = -\lambda t + C$$

$$\text{Require } P(0,t_1) = 1$$

$$0 = -\lambda t_1 + C$$

$$C = \lambda t_1 \text{ (Alternatively put the limits in the integral)}$$

$$\ln|P(0,t)| = -\lambda(t - t_1)$$

$$P(0,t) = e^{-\lambda(t-t_1)}$$

Once the muons enter the detector and come to rest, they then decay into electrons/positrons at some later time t_2 giving off the second scintillation flash. Thus the probability that a muon causes a scintillation flash at time t_1, doesn't decay until time t_2 and then decays within the small time interval between t_2 and $t_2 + dt$ causing a second scintillation flash is given by

$$dP_e(t) = \lambda e^{-\lambda(t_2 - t_1)} dt \tag{5}$$

In the experiment we are measuring the time between the two scintillation flashes $t_2 - t_1$ for each detected muon event. We, therefore, expect this quantity to follow an exponential distribution given by equation 5. By measuring many many muon events we can thus construct such a distribution from our observations and determine the mean muon lifetime $\tau_\mu = 1/\lambda$ by fitting the theoretical distribution to our results.

4 Method

The following are practical steps when using the equipment:

1. Check that all equipment is switched off at the mains power supply.

2. Plug in the USB cable to the PC and back of the Muon Physics unit.

3. From the rear of the Muon Physics box connect the DC Output to the DC Input on the PMT-Scintillator.

4. Take a coax cable, connect one end to the PMT Output on the PMT-Scintillator and the other to the PMT input on the Muon Physics box.

5. The other two coax cables should be connected between the Oscilloscope and Muon Physics box, taking note of which signal corresponds to the colour on the screen on the scope. This is often identified by the colour of the buttons next to the signal input.

6. Turn on the PC, log in and turn on the power supplies to the other equipment.
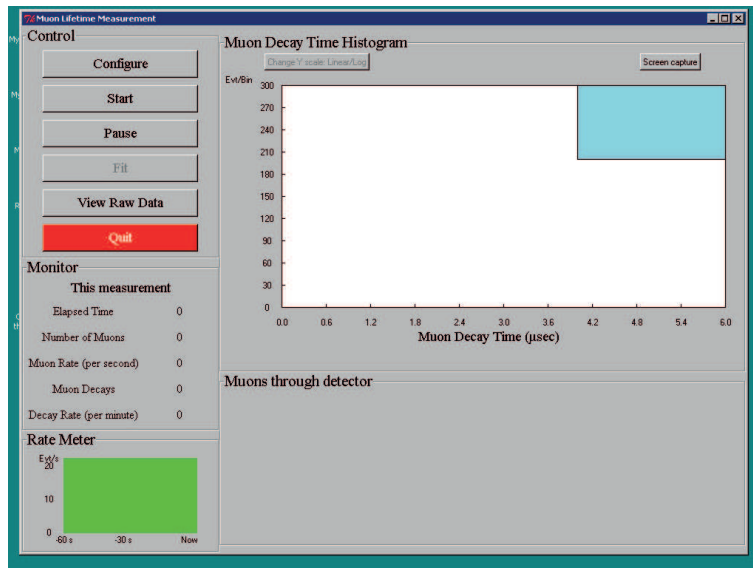
Figure 4: GUI of the data analysis software.

7. Check that the Pulse switch is set to 'off' on the PMT-Scintillator as this is used for calibration.

8. Adjust the HV to around 8 on the PMT-Scintillator, this adjusts the voltage of the PMT.

9. On the Muon Physics Box there is a knob to adjust the cut-off threshold for background radiation. One will hear that when the audio is on, increasing the threshold reduces the number of clicks per second. It is advised to have this at between 4-6 and may require help from your supervisor. If this is set incorrectly it can ruin any datasets you take as too much background is included. This can subsequently give a deviation from the true value.

10. On the PC: Open the folder named muon data, then open the application called muon. The graphical user interface (GUI) should look like that in figure 4. Select 'Configure' and choose one of the comN buttons where $N = 1, 2, 3$ or 4, select 'save/exit' and press 'Start' upon returning to the main GUI.

11. If the 'Number of Muons' does not change after a 20 second period, close the application and repeat the previous set with a different port selected. If successful, one should observe the 'Number of Muons' increase with time and the histogram gradually being filled with data.

12. It is advised to run the experiment for as long as possible as this is a number counting experiment based on Poisson statistics and with more data the statistical error is reduced.

13. After running for a prolonged period of time, stop the experiment with the 'stop' button, then press the 'fit' button and take a screen shot of the GUI.

14. Export the raw data for further analysis (see section 5).

5 Analysis

While the GUI gives us an initial estimate of the muon lifetime, we need to perform a more sophisticated analysis e.g. in order to quantify the statistical errors in our lifetime estimate.

Scientists use a number of packages to do such analysis like MatLAB, Python, R etc. If you do not have access to such software you can use Plotly (http://plot.ly) which is an online plotting and analysis tool suitable for the purposes of this project. Note you will need to sign up to an account if you want to save or export your graphs.

You should import your data into the analysis package and construct a histogram of the number of events against the decay time $t_2 - t_1$. Note that you will need to set the width of your histograms, if this is too small your distribution will be very noisy whereas if it is too large it will smear out the results too much (see the link in section 8 for more on how to pick the optimal bin size). You will also need to set the bin limits to remove any anomalous bins.

Once you have a histogram you should add error bars to it. This might seem counter-intuitive initially as the histogram shows the actual number of muon events within a certain decay time range. However, as we mentioned the decay is a probabilistic process so if we were to repeat the experiment we may get a different number of events. Poisson statistics (used for large numbers of rare events) tell us that the standard deviation of the number of events should go as $\sqrt{N_k}$ where N_k is the number of events within bin k. The error bars thus represent the spread of possible numbers of events in each bin.

> **Exercise:** Flip a coin M times (e.g. 10) and count the number of heads. Repeat this a few times (e.g. 5) and calculate the mean and standard deviation of the number of heads in each try.
>
> Is the mean number of heads consistent with the expected value pM, where $p = 0.5$ is the probability of landing heads, and the standard deviation around $\sqrt{pM}$?
>
> Retry the experiment this time with a dice, i.e. changing p, and compare again with the theoretical result.

To get an estimate of the muon lifetime you will need to perform a least-squares fit to your histogram. This is an iterative procedure (like the Newton-Raphson method you may have encountered) which attempts to minimise the difference between the model fit and the actual observations. You should use a fit function of the form

$$y = a + be^{cx} \tag{6}$$

You'll note that this is slightly different from equation 5 as it also includes a constant background term a. To perform the fit you will need to give the fitting program an initial guess for all three parameters. Your earlier results from the GUI might be useful here, note that if the initial guess is very poor the algorithm may not be able to find a good fit to the data. This procedure will give you estimates and their associated statistical errors for the three parameters in equation 6, which you can then use to arrive at a value and error for the mean lifetime of muons.

6 Discussion

Here we raise some points for you to think about after performing your initial analysis:

- Is your value of the mean lifetime of muons consistent (to within errors) of the current published value? If not, then why?
  The lifetime they determine should be slightly smaller than the published value of the mean lifetime of muons in free space of $2.1969811 \pm 0.0000022\,\mu s$ and, depending on their errors, may not be consistent. This is because the muons are in a medium which presents an alternative decay path for the negative muons: muon capture (discussed below). This

essentially increases the probability of decay for the negative muons $\lambda^-_{eff} = \lambda + \lambda_c$. The overall effective lifetime is hence decreased and depends on the ratio of positive to negative muons.

- How dependent are your results on the analysis? e.g. does the lifetime significantly change if you slightly adjust your bins?
  If they use a sensible bin size then this shouldn't be the case, but it's good to see how the results and errors are affected by the bin size. Note they may want to plot histograms with different bin sizes on the same graph. If so to make them comparable they can change the y-axis from being the number of events in the bin to the probability density ($p/\Delta x$ in Plotly) so that the histograms/fits should lie on top of one another.

- Are there any other routes through which muons could decay in the detector? If so could this effect be taken into account?
  Negative muons can interact with the protons in the plastic scintillator material nuclei $\mu^- + p \to n + \nu_\mu$. The likelihood of this interaction goes as Z^4, where Z is the atomic number of the nuclei, and the lifetime for the scintillator should be close to that of carbon $\tau_c = 2.043 \pm 0.003\,\mu s$. To take this into account you need to know the ratio of positive to negative muons. The students could perhaps look this up or instead use their results compared to the published lifetime in free space to estimate this ratio. This latter activity leads into one of the proposed research extensions (#3).

- Are there any other ways an event could register in the detector not due to a single muon decay? What is the likelihood of this?
  If the detector has been properly calibrated it should only be registering predominantly muons, though during large geomagnetic storms it is possible that some charged particles either from e.g. radiation belts or the Sun could perhaps be detected. This is more likely at higher latitudes. It is also possible two scintillation flashes could occur due to two muons entering the detector at nearly the same time. This is unlikely given the flux of cosmic ray muons and the cutoff time has been set so this rarely happens. This is discussed further in the 'Muon Physics' Introduction linked to in section 8.

- Given the errors involved, are you using a suitable number of significant figures when publishing your results?
  Students have a tendency to quote all the decimal places from their analysis, even when the errors are large. Usually 2 significant figures for the errors are sufficient and thus the quoted results should be displayed to the same level of precision as the errors.

7   Research

Now that you've gotten to grips with using the detector and undertaking project work, it is up to you to continue this work with your own independently motivated research. It is imperative that you perform a literature review before continuing, reading up on and around the subject of muons, cosmic rays etc., to make sure you are up to date with current results and understanding in your field of study. Here we merely suggest some avenues you may wish to explore in your own research:

1. Which direction(s) do muons come from? Does this tell us anything about cosmic ray generation mechanisms?

2. Does the flux of cosmic rays into the atmosphere vary with time? What factors affect the measured muon flux on the ground?

3. Are there differences in the properties and/or numbers of negatively charged muons and their positively charged antiparticles in our atmosphere?

4. How do special relativistic effects, such as time dilation, factor in to cosmic ray muon observations?

5. How can we arrive at measures of the coupling constant of the Weak interaction and/or the Higgs field vacuum expectation value in the Standard Model by observing muons?

6. How can we measure the energy distribution of cosmic ray muons using the detector?

You may use one of these topics as inspiration or come up with one of your own to research. Be sure to read around your chosen topic and design a suitable experiment using the detector that will address your research question.

8 Useful links

Atmospheric Muon Basics: http://hyperphysics.phy-astr.gsu.edu/hbase/particles/muonatm.html
Atmospheric Muon Fluxes and Energies: http://arxiv.org/pdf/1208.1171.pdf
'Muon Physics' Introduction http://www.teachspin.com/newsletters/Muon%20ConceptualIntroduction.pdf
'Muon Physics' Manual: http://web.phys.ntu.edu.tw/asc/FunPhysExp/ModernPhys/exp/muonphysics.pdf
Cosmic Rays: http://pdg.lbl.gov/2013/reviews/rpp2013-rev-cosmic-rays.pdf
Cosmic Ray Muon Angles and Relativity: http://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm
Histogram Bin Widths: http://toyoizumilab.brain.riken.jp/hideaki/res/histogram.html
Histogram Error Bars: http://arxiv.org/pdf/1112.2593v3.pdf